Attendance Problems. Simplify each radical.

1. $\sqrt{12}$ **2.** $\sqrt{50}$ **3.** $\sqrt{75}$

Find the distance between each pair of points. Write your answer in simplest radical form.

4. (1, 6) and (-2, 0) **5.** (-7, -1) and (-1, 5)

- I can apply similarity properties in the coordinate plane.
- I can use coordinate proof to prove figures similar.

Vocabulary	
dilation	scale factor

Common Core

CC.9-12.G.CO.2 Represent transformations in the plane. Compare transformations that preserve distance and angle to those that do not.

CC.9-12.G.SRT.1 Verify the properties of dilations given by a center and a scale factor:

a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.

b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

A **dilation** is a transformation that changes the size of a figure but not its shape. The preimage and the image are always similar. A **scale factor** describes how much the figure is enlarged or reduced. For a dilation with scale factor k, you can find the image of a point by multiplying each coordinate by k: $(a, b) \to (ka, kb)$.

Helpful Hint

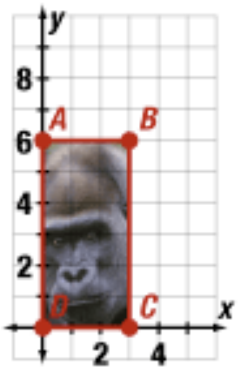
If the scale factor of a dilation is greater than 1 ($k > 1$), it is an *enlargement*. If the scale factor is less than 1 ($k < 1$), it is a *reduction*.

Q: What do you get when you double a triangle?
A: A try-again angle!

"Conduct is more convincing than language."—*18th Century Cleric, John Woolman*

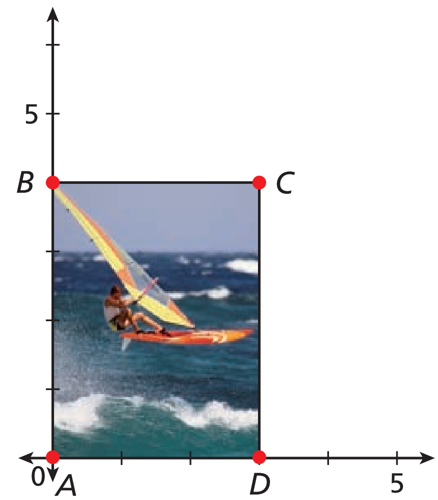
Refer to video example 1.

The figure shows the position of a JPEG photo. Draw the border of the photo after a dilation with scale factor $\frac{4}{3}$.



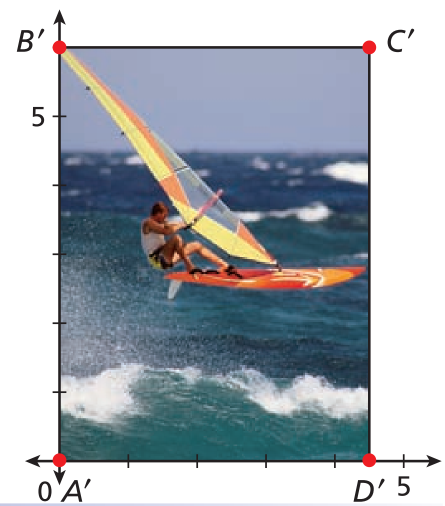
1 Computer Graphics Application

The figure shows the position of a JPEG photo. Draw the border of the photo after a dilation with scale factor $\frac{3}{2}$.



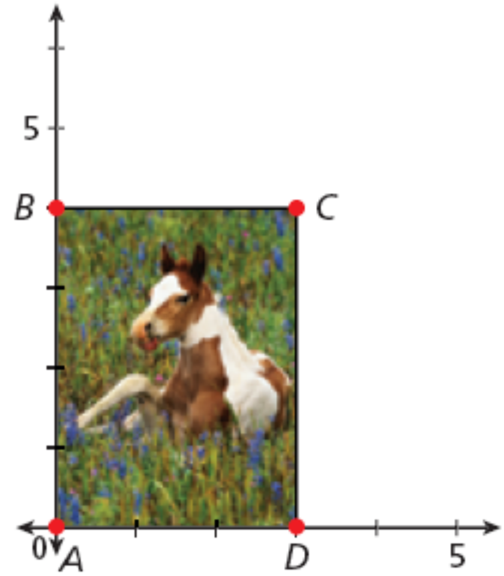
Step 1 Multiply the vertices of the photo $A(0, 0)$, $B(0, 4)$, $C(3, 4)$, and $D(3, 0)$ by $\frac{3}{2}$.

Rectangle $ABCD$		**Rectangle $A'B'C'D'$**
$A(0, 0) \rightarrow$	$A'\left(0 \cdot \frac{3}{2}, 0 \cdot \frac{3}{2}\right) \rightarrow$	$A'(0, 0)$
$B(0, 4) \rightarrow$	$B'\left(0 \cdot \frac{3}{2}, 4 \cdot \frac{3}{2}\right) \rightarrow$	$B'(0, 6)$
$C(3, 4) \rightarrow$	$C'\left(3 \cdot \frac{3}{2}, 4 \cdot \frac{3}{2}\right) \rightarrow$	$C'(4.5, 6)$
$D(3, 0) \rightarrow$	$D'\left(3 \cdot \frac{3}{2}, 0 \cdot \frac{3}{2}\right) \rightarrow$	$D'(4.5, 0)$

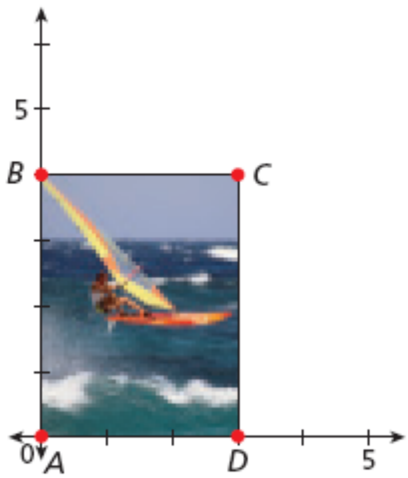


Step 2 Plot points $A'(0, 0)$, $B'(0, 6)$, $C'(4.5, 6)$, and $D'(4.5, 0)$. Draw the rectangle.

<u>Example 1</u>. Draw the border of the photo after a dilation with scale factor $\frac{5}{2}$.

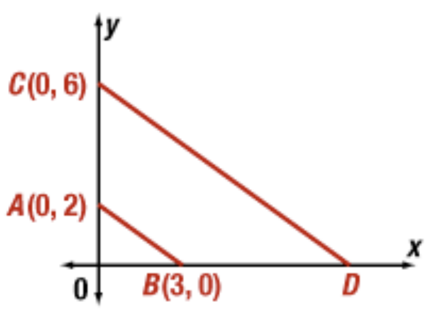


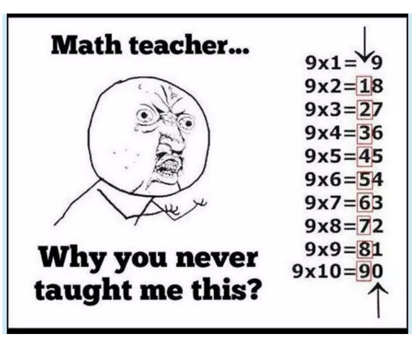
6. **<u>Guided Practice.</u>** Draw the border of the original photo after a dilation with scale factor $\frac{1}{2}$.



Refer to video Example 2.

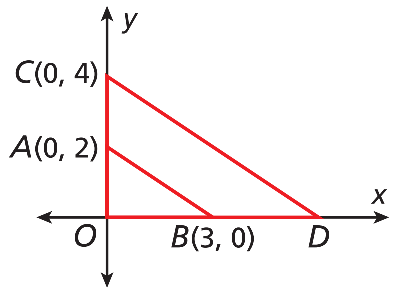
Given that $\triangle AOB \sim \triangle COD$, find the coordinates of D and the scale factor.





2 Finding Coordinates of Similar Triangles

Given that $\triangle AOB \sim \triangle COD$, find the coordinates of D and the scale factor.



Since $\triangle AOB \sim \triangle COD$,

$$\frac{AO}{CO} = \frac{OB}{OD}$$

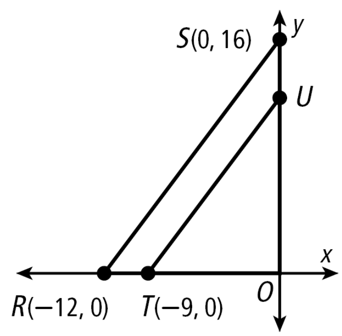
$$\frac{2}{4} = \frac{3}{OD}$$ *Substitute 2 for AO, 4 for CO, and 3 for OB.*

$$2OD = 12$$ *Cross Products Prop.*

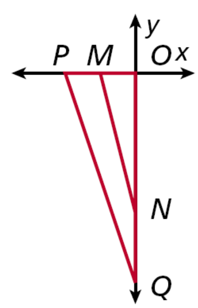
$$OD = 6$$ *Divide both sides by 2.*

D lies on the x-axis, so its y-coordinate is 0. Since $OD = 6$, its x-coordinate must be 6. The coordinates of D are $(6, 0)$.
$(3, 0) \rightarrow (3 \cdot 2, 0 \cdot 2) \rightarrow (6, 0)$, so the scale factor is 2.

Example 2. Given that $\Delta TUO \sim \Delta RSO$, find the coordinates of U and the scale factor.



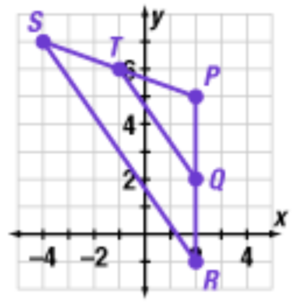
7. Guided Practice. Given that $\Delta MON \sim \Delta POQ$ and coordinates $P(-15, 0)$, $M(-10, 0)$, and $Q(0, -30)$, find the coordinates of N and the scale factor.



<u>Refer to video Example 3</u>.

Given:
$P(2, 5)$, $Q(2, 2)$, $R(2, -1)$, $S(-4, 7)$, and $T(-1, 6)$

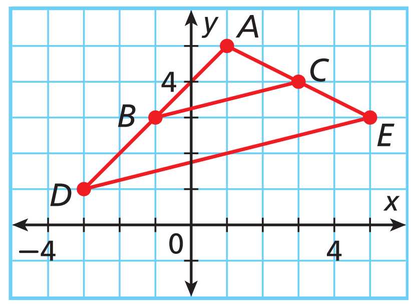
Prove:
$\triangle PQT \sim \triangle PRS$



3 Proving Triangles Are Similar

Given: $A(1, 5)$, $B(-1, 3)$, $C(3, 4)$, $D(-3, 1)$, and $E(5, 3)$

Prove: $\triangle ABC \sim \triangle ADE$



Step 1 Plot the points and draw the triangles.

Step 2 Use the Distance Formula to find the side lengths.

$$AB = \sqrt{(-1-1)^2 + (3-5)^2} = \sqrt{8} = 2\sqrt{2}$$

$$AC = \sqrt{(3-1)^2 + (4-5)^2} = \sqrt{5}$$

$$AD = \sqrt{(-3-1)^2 + (1-5)^2} = \sqrt{32} = 4\sqrt{2}$$

$$AE = \sqrt{(5-1)^2 + (3-5)^2} = \sqrt{20} = 2\sqrt{5}$$

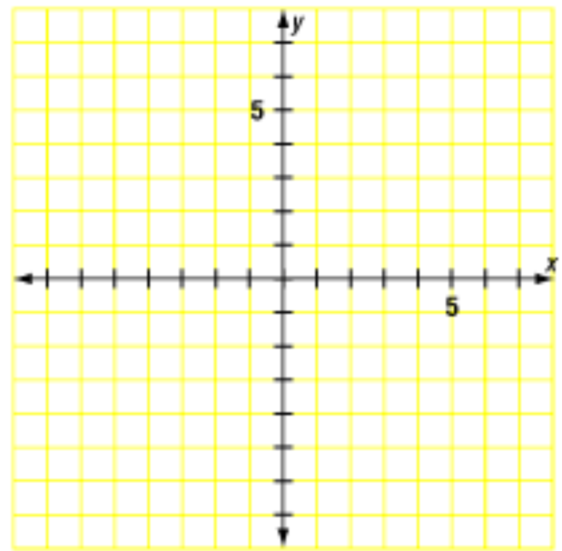
Step 3 Find the similarity ratio.

$$\frac{AB}{AD} = \frac{2\sqrt{2}}{4\sqrt{2}} = \frac{2}{4} = \frac{1}{2}$$

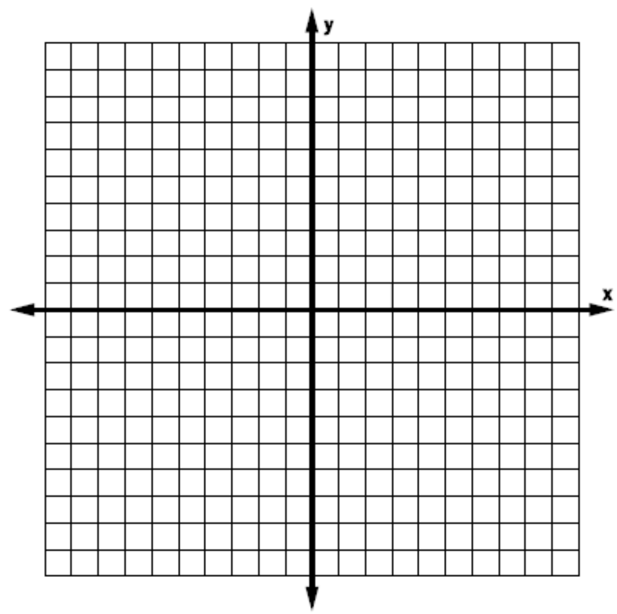
$$\frac{AC}{AE} = \frac{\sqrt{5}}{2\sqrt{5}} = \frac{1}{2}$$

Since $\frac{AB}{AD} = \frac{AC}{AE}$ and $\angle A \cong \angle A$ by the Reflexive Property, $\triangle ABC \sim \triangle ADE$ by SAS $\sim$.

<u>Example 3</u>. Given: $E(-2, -6)$, $F(-3, -2)$, $G(2, -2)$, $H(-4, 2)$, and $J(6, 2)$.
Prove: $\Delta EHJ \sim \Delta EFG$.



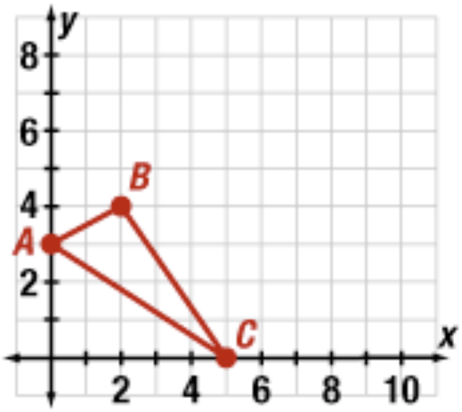
8. **<u>Guided Practice.</u>**
Given: $R(-2, 0)$, $S(-3, 1)$, $T(0, 1)$, $U(-5, 3)$, and $V(4, 3)$.
Prove: $\Delta RST \sim \Delta RUV$



<u>7-6 Dilations and Similarity in the Coordinate Plane</u>: *(p 513)* 10-14.

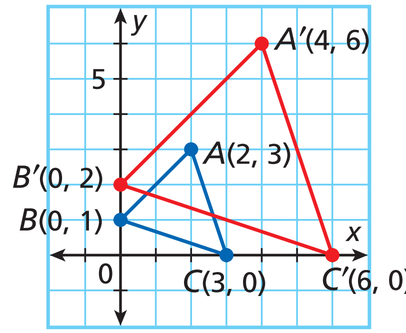
<u>Refer to video Example 4</u>.

Graph the image of $\triangle ABC$ after a dilation with scale factor 2.
Verify that $\triangle A'B'C' \sim \triangle ABC$.



4 Using the SSS Similarity Theorem

Graph the image of $\triangle ABC$ after a dilation with scale factor 2. Verify that $\triangle A'B'C' \sim \triangle ABC$.



Step 1 Multiply each coordinate by 2 to find the coordinates of the vertices of $\triangle A'B'C'$.

$$A(2, 3) \rightarrow A'(2 \cdot 2, 3 \cdot 2) = A'(4, 6)$$
$$B(0, 1) \rightarrow B'(0 \cdot 2, 1 \cdot 2) = B'(0, 2)$$
$$C(3, 0) \rightarrow C'(3 \cdot 2, 0 \cdot 2) = C'(6, 0)$$

Step 2 Graph $\triangle A'B'C'$.

Step 3 Use the Distance Formula to find the side lengths.

$$AB = \sqrt{(2-0)^2 + (3-1)^2} = \sqrt{8} = 2\sqrt{2} \qquad A'B' = \sqrt{(4-0)^2 + (6-2)^2} = \sqrt{32} = 4\sqrt{2}$$

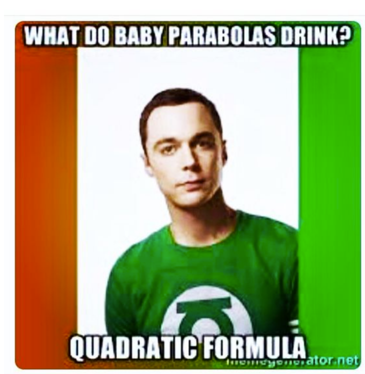
$$BC = \sqrt{(3-0)^2 + (0-1)^2} = \sqrt{10} \qquad B'C' = \sqrt{(6-0)^2 + (0-2)^2} = \sqrt{40} = 2\sqrt{10}$$

$$AC = \sqrt{(3-2)^2 + (0-3)^2} = \sqrt{10} \qquad A'C' = \sqrt{(6-4)^2 + (0-6)^2} = \sqrt{40} = 2\sqrt{10}$$

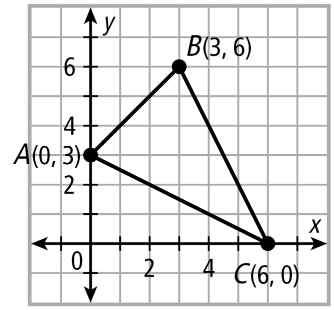
Step 4 Find the similarity ratio.

$$\frac{A'B'}{AB} = \frac{4\sqrt{2}}{2\sqrt{2}} = 2, \quad \frac{B'C'}{BC} = \frac{2\sqrt{10}}{\sqrt{10}} = 2, \quad \frac{A'C'}{AC} = \frac{2\sqrt{10}}{\sqrt{10}} = 2$$

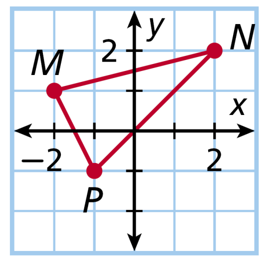
Since $\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC}$, $\triangle ABC \sim \triangle A'B'C'$ by SSS $\sim$.



Example 4. Graph the image of ΔABC after a dilation with scale factor $\frac{2}{3}$.
Verify that $\Delta A'B'C' \sim \Delta ABC$.



9. Guided Practice. Graph the image of ΔMNP after a dilation with scale factor 3. Verify that $\Delta M'N'P' \sim \Delta MNP$.



7-6 Dilations and Similarity in the Coordinate Plane

- *(p 513)* 10-16, 18, 20.
- 7B Ready to Go On & posttests.